An Identification Genetic Algorithm for the Duffing's Oscillator

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ABSTRACT

An off-line method to recover the whole unknown parameter set of the Duffing's oscillator by means of a genetic algorithm is presented in this paper. The fact that the system is observable and constructible with respect to a suitable output (measurable variable) allows obtaining an integral parametrization of the output. Based on this integral parametrization of the output, that depends on the unknown parameters, a random estimation of the output can be generated by finding many solutions. The unknown parameters are contained in a bounded set. This random estimation is obtained provided that the error between to the actual output and the estimated output minimizes a quadratic function of the errors. The minimization problem and the random estimations of the output are formulated directly in terms of a genetic algorithm. A population of chromosomes is codified with the parameters of the Duffing's oscillator system. A fitness function is established to evaluate the chromosomes, in such a manner that it minimizes a quadratic function of the errors. The population of chromosomes evolves until a fitness average threshold is obtained. This method is numerically feasible and easy to implement in a digital computer.

Keywords
Mechanical Oscillator, Chaos, Genetic Algorithms, Reconstruction.

INTRODUCTION

The reconstruction and identification of chaotic attractors from one or more suitable states is one of the most challenging problems in chaos theory and its applications. Thus, several methods for reconstructing and identifying chaotic systems have been proposed in the literature (for a deeper treatment of those methods, the reader is referred to [3], [6] and [7]). In these works, the authors have applied control theory to design state observers and system identification schemes for recovering the missing variables and the unknown parameters. The other important approach is based on the well-known Takens' Theorem (see [1], [12], [13], [14], [15]). This methodology consists in analyzing the observed time series from a nonlinear system for reconstructing a time delay of a phase space, in which it is possible to analyze the attractor. This is carried out by using time delayed values of an observed scalar quantity as coordinates for the phase space. Roughly speaking, vector state $y(n)$ constructed as:

$$y(n) = [x(n), x(n+T), \ldots, x(n+(d-1)T)]$$

can be estimated from a set of observations. Here x is the observed variable, T s the time delay and d is the embedding dimension (see [22]). The last approach is based on soft computing, as proposed in [9], [10], [11], [3]. In those works, the unknown chaotic system is viewed as a black box belonging to a class of nonlinearities. Therefore, the dynamic neural network can be used to recover the unknown parameters.

In this paper, we propose a simple and efficient approach for revealing all the unknown parameters and estimating the velocity state for the Duffing's system by means of a genetic algorithm (GA). The purpose of a GA in any application is to evolve a chromosome population that codifies several possible solutions of the problem using genetic operators like *selection*, *crossover* and *mutation*. The goal of GA is optimization of a fitness or cost function that depends on the problem to solve. In our case, the main idea consists on minimizing the norm of a quadratic function, which depends on the unknown parameters and successive integrations of a suitable output (the position of the Duffing's system). The integral parametrization of the output of the Duffing's oscillator is necessary for shaping the positive function that will be minimized. The minimum of the function is reached when the actual parameter values are attained. The proposed methodology differs from the one described in [15] and [18] because we avoid the necessity of computing the derivatives of the measurable variable states (by using many pairs of timepoints and averaging the estimates), instead of, we apply successive integrations of the output. It is worth to mention that our method also differs from others

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since it can estimate the parameter w (which is the force frequency).

The rest of the paper is organized as follows. Section 1 contains a brief introduction to Duffing's system. Including an integral parametrization of the output and the parameters identification. Section 2 presents the numerical implementation of the used GA and describes the results obtained, while Section 3 is devoted to conclusions and suggestions for further research.

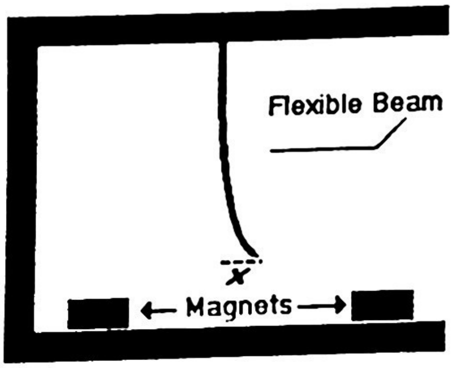

Fig.1 Duffing's oscillator

I. Duffing's Mechanical Oscillator

Figure 1 depicts a diagram of Duffing's system. This oscillator is formed of a flexible steel beam tied to the top center of a box. On either side of the bottom of the box are two electromagnets. When an alternating current excites the electromagnets, magnetic field makes the bar to move. The non-linear model, which can be found in [1] and [17], is given by:

$$\begin{aligned} \dot{x} &= v \\ \dot{v} &= -p_1 v - p_3 x^3 + p_2 x + A\cos(\omega t) \end{aligned} \qquad (1)$$

The horizontal displacement from the vertical at the lower extremity of the beam is measured by x. The magnitude of the forcing function is denoted with A, the forcing frequency is ?, the damping coefficient is p_1, and the fixed constants, which are related to the non-linear stiffness function of the beam, are p_2 and p_3.

It is known that the system has a chaotic behavior [22] for the fixed values of parameters in a neighborhood of $\{p_1 = 0.4, p_2 = -1.1, p_3 = 1, A = 2.1, \omega = 1.8\}$.

A. *An integral parametrization of the output*

In this section, we introduce a simple integral parametrization for Duffing's system. This parametrization allows us for building a function that depends on the unknown parameters and on successive integrations of the measurable (output).

Let us consider equation () and let us take as $y = x$ (*i.e.* the position is known). Then, the velocity state $v = y$ can be obtained by means of an integra parametrization as follows:

$$v = v_0 - p_1(y - y_0) - p_3\int y^3 + p_2\int y + \frac{A}{\omega}(\sin(\omega t) - \sin(\omega t_0))$$

where y_0 and v_0 stand for the initial conditions states $x(t_0)$ and $v(t_0)$ respectively. Symbol denotes the quantity $\left(\int_{t_0}^{t} x(\tau_1)d\tau_1\right)$ Iterat integrals, such as $\left(\iint x\right)$ are also used without integration variables and integration limits.

Note that the integral parametrization given equation (2), allows us to measure the velocity sta v, as a function of the output, y modulo the conditions.

Integrating equation (2) once again with respect time from initial time t_0 to the final time t we have t following iterated integral equation:

$$\begin{aligned} y(t) &= y_0 + v_0(t - t_0) + p_1\left[y_0(t - t_0) - \int y\right] \\ &\quad + p_2\iint y - p_3\iint y^3 \\ &- A\left[\frac{1}{\omega^2}(\cos(\omega t) - \cos(w t_0)) + \frac{1}{\omega}\sin(\omega t_0)(t\right. \end{aligned}$$

From (3) we conclude that the system is observat and constructible with respect to the output [24]). The last equation contains the information for recovering the set of parameters. the reconstruction process an error is generated. that must be minimized as a quadratic function parameters $\{v_0, p_1, p_2, p_3, A, \omega\}$. In the followi section we present such quadratic function.

B. *Parameters identification for the Duffing System*

It is known that parameter recovering of a based on the measurements of one or variables leads to an optimization problem characterized by its ill-posed nature in the Hadama sense (see [23] for more details), since a solution cannot be obtained. Usually, the issue be solved using some variant of Newton's method conjugated gradient algorithm or even quadratic method (see [25]). In this section we the problem as follows. First, a parametric estimator for the output $\hat{y}$ of (3) is proposed. Next, a quadratic error functions between y and $\hat{y}$ is created. Finally, a simple GA is applied to find an optimal solution.

We begin by establishing the identification problem for the Duffing's system:

Let $q = [q_0, q_1, q_2, q_3, q_4, q_5]$ be the vector of unknown parameters where $q_i \in R$ for $i = 0,1,...,5$, and let $\hat{y}(\cdot)$ be the following estimation output function:

$$\hat{y}(t,q) \equiv y_0 + q_0\phi_1(t) + q_1[y_0\phi_1(t) + \phi_2(t)] + q_2\phi_3(t) + q_3\phi_4(t) - q_4 f(q_4, q_5, t) \quad (4)$$

where the set of variables $\{\phi_1(t), \phi_2(t), \phi_3(t), \phi_4(t)\}$ is an iterated integral function of the measurable output, y which is defined by:

$$\phi_1(t) = t - t_0; \quad \phi_2(t) = -\int y; \quad \phi_3(t) = \iint y; \quad \phi_4(t) = -\iint y^3 \quad (5)$$

and finally $f(A, \omega, t)$ denotes the time dependent function given by

$$f(q_4, q_5, t) = \frac{q_4}{q_5^2}(\cos(q_5 t) - 1) + \frac{q_4}{q_5}\sin(q_5 t_0)(t - t_0)$$

Now, consider the following estimation error defined as:

$$e(t) = y(t) - \hat{y}(t,q) \quad (6)$$

Note that if vector q takes the real values of Duffing's system given by $q_r = [v_0, p_1, p_2, p_3, A, \omega]$ then $e(t) = 0$ for any time t. This means that the problem of finding the vector of unknown parameter $q \in R^6$ is clearly equivalent to solving the following unconstrained minimization problem:

$$\min_{q \in R^6} \sum_{k=1}^{n} e^2(kT) \quad (7)$$

where T is the sampling time and n is the total number of samples.

To find the minimum of the last expression, it is necessary to introduce the following basic assumptions:

A.1 The strings of outputs $y(t-kt)$ for a fixed delay $t > 0$ and k={0,1,...,n} are available for any time t, such that $t > kt$.

A.2 The auxiliary functions $\phi_i(t - k\tau)$ for a fixed delay $t > 0$ and k={0,1,...,n} defined in (5) can be stored and computed.

A.3 The set of parameters of the non-linear system (1) belongs to a neighborhood of $\{p_1 = 0.4, p_2 = -1.1, p_3 = 1, A = 2.1, \omega = 1.8\}$. Therefore, variable y belongs to a chaotic attractor.

The problem of finding q such that expression (7) is minimized, can be solved by a numerical implementation of the well-known Newton's method or some variations of it. However, instead of Newton's method, we employ a GA which avoids the possibility of falling into a local minimum. The GA creates a population of q in a stochastic fashion according to some basic rules (see [19], [20], [21]). Then, it selects the element that produces the smallest error in (7).

Remark 1 The objective of all optimization problems is to find a minimum or maximum objective function value. Considering minimum values, usually a problem may have more than one minimum objective function value. There are many traditional deterministic algorithms available to solve optimization problems for a local minimum. Some of these methods include the descent gradient techniques. These methods require the evaluation of gradient information in order to solve the problem. Gradient evaluations can become difficult and time consuming when complex objective functions are present. These methods always look for the closest minimum, without regarding it is a local or global one.

II. NUMERICAL IMPLEMETATION OF THE GA

Computer simulations have been carried out in order to estimate the unknown parameters $\{p_1, p_2, p_3, A, \omega\}$ and initial state v_0 for the Duffing's system given in (1). The numerical program was implemented by using the fourth-order Runge-Kutta algorithm. The computation was performed with 8 decimal digit numbers. To obtain a good performance, the step size in the numerical method was set to 0.001. The parameter values were taken as $p_1 = 0.35$, $p_2 = -1.0145$, $p_3 = 0.9567$ $A = 2.15$ and $\alpha = 1.8931$. The sampling time was selected as $T=0.25$ sec. and the number of samples was chosen as $n=20$. The initial conditions where taken as $y(0)=0.3$ and $\dot{y}(0)=-2.3$ respectively.

Now, we describe in a general fashion how the GA is employed to minimize the function given in (7).

1. Individuals in the GA are vectors (in R^6) of the form
$$q_i = [q_{0,i}, q_{1,i}, q_{2,i}, q_{3,i}, q_{4,i}, q_{5,i}]$$
It can be seen that the GA is a real-coded one (as opposed to a binary coded one).
2. The initial population, P_0, contains 500 individuals, while subsequent populations, P_j, consist of 100 individuals. This allows to perform a wider search with the initial population while concentrating on more specific regions afterwards.
3. The best individual, q_1 (evidently ranked 1st), in generation P_j is passed on to generation P_{j+1}, with no change.
4. Several steps are involved in the creation of generation P_{j+1}, they are as follows: a) selection; b) crossover; c) mutation.

   a) To accomplish selection, each individual in P_j is assigned a probability which is calculated linearly according to it's ranking in the whole population . Selection of individuals is made by generating random numbers in [0,1] (say a_i) and comparing them to the accumulated probability, $Ap(q_i)$, of each individual. Individual q_i is selected to be part of P'_{j+1} when $a_i = Ap(q_i)$. This is the well-known *roulette selection* scheme.

   b) The crossover algorithm used in this GA is a slight modification of the *flat crossover* (or *arithmetic crossover*) operator (see [4] ,[21],[5]). An "offspring" $h=[h_0,h_1,h_2,h_3,h_4,h_5]$ is generated as
$$h_i = \beta \cdot q_{i,1} + (1-\beta)\cdot q_{i,2}$$
   from "parents"
$$q_1 = [q_{0,1}, q_{1,1}, q_{2,1}, q_{3,1}, q_{4,1}, q_{5,1}]$$
$$q_2 = [q_{0,2}, q_{1,2}, q_{2,2}, q_{3,2}, q_{4,2}, q_{5,2}]$$
   where q_1 is a better individual that q_2 (i.e. q_1 makes the error function smaller than q_2 does).and β is a random number chosen uniformly from the interval [0.5,1]. This interval is used in order to weight as more "influential" the information carried by the best of the parents. This process is repeated until there are 99 "offspring" (q_1 passes on unchanged).

   c) The mutation algorithm consists in randomly changing a component of 50% of the individuals created at the last step. Changes are done within the vicinities specified below. This is the final step creating generation P_{j+1}.
5. The "cost" of each individual was calculated via $\sum_{k=1}^{n} e^2(kT)$ where T = 0.25 and n = The algorithm stops when the best individual reaches a "cost" of 10^{-9}.or less
6. Components of vector $q=[q_0,q_1,q_2,q_3,q_4,q_5]$ were searched in vicinities of radius 2.5 for $q_1,\ldots,q_5$ and radius 5 for q_0. Centered
$$\bar{q} = \left[\frac{y_T - y_0}{T}, 0.4, -1.1, 1, 2.1, 1.8\right].$$

In the following figures we present the obtained results after applying the previously described In figure 2, four signals are displayed: measurable output and three reconstructed signals These signals correspond to the best individual generations 1, 200, and 22793 (last generation) Note that the last one cannot be distinguished the original output. This means that the error in estimated parameter is almost zero.

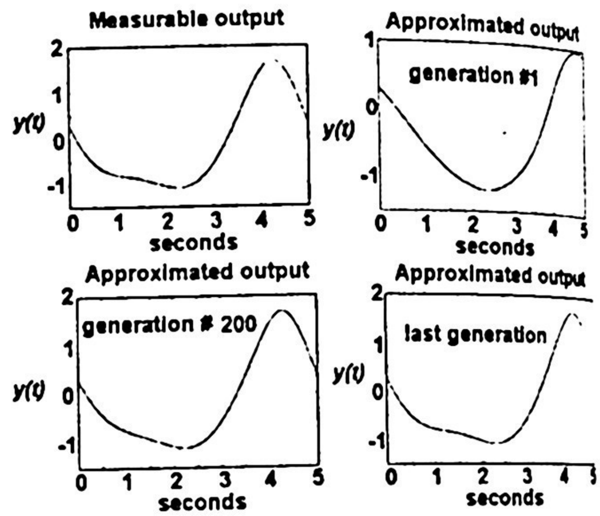

Figure 2. Measurable output and three reconstructed signals.

Figures 3-5 show the evolution process of the initial state v_0 and of parameters p_1, p_2, p_3, A, ω respectively through generations.

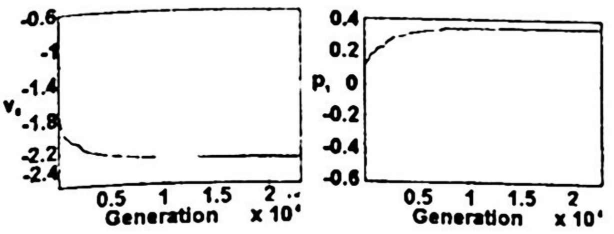

Figure 3. Evolution process of initial state v_0, and parameter p_1.

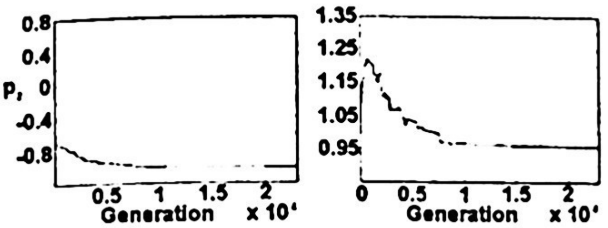

Figure 4. Evolution process parameters p_2 and p_3.

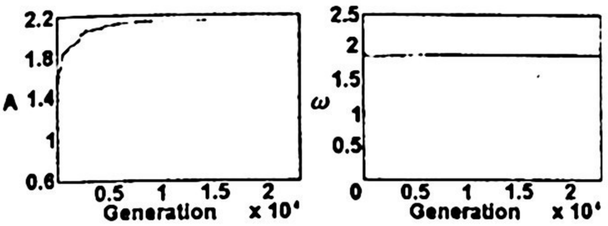

Figure 5. Evolution process of parameters A and ?.

Finally, in Figure 6 we show the behavior of expression (7); the process of error minimization. It can be seen that the error tends to zero when the generation number increases, therefore, the reconstructed output is nearly the original output. Evidently, this implies that the actual and estimated parameters are very close, as shown in Table 1.

III. Conclusions

A method for recovering parameters and estimating the velocity state of the Duffing's oscillator was proposed. We exploit the fact that the system is observable and constructible with respect to a measurable output, which is the flexible beam position. This property permits us to build an iterated integral equation of the available output, which contains the needed information for recovering the absent state and the unknown parameters. Based on the iterated integral equation, we estimate the output (defined in (4)) assuming that physical parameters of the system are unknown. The main idea is to minimize the difference between the available output and the estimated output, as described in (7). The minimization process is carried out applying the GA. This approach was validated by means of numerical experiments, in which the quadratic error was efficiently minimized, and, therefore, the parameters and the unknown state were estimated in a satisfactory way.

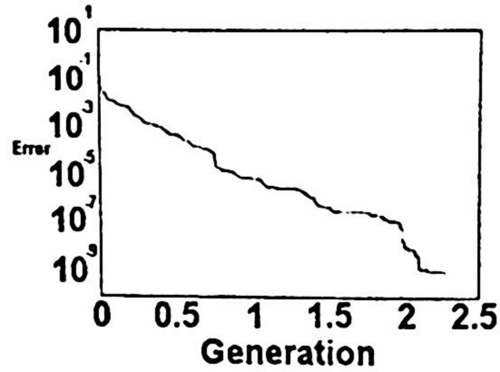

Figure 6. Error minimization process.

Ge	v_0	p_1	p_2	p_3	A	?
1	-0.7389	-0.5700	0.6310	0.8817	0.6647	0.3456
3	-1.1938	-0.1472	-0.3600	0.9749	1.2590	2.2151
9	-1.7830	0.1580	-0.6116	1.0864	1.7610	1.9843
28	-1.7169	0.0996	-0.6326	1.0769	1.4965	1.9636
86	-1.7	0.1237	-0.6869	1.0917	1.5967	1.9126
264	-1.9080	0.1402	-0.7073	1.1497	1.6746	1.8787
804	-2.0347	0.1839	-0.7342	1.2125	1.6539	1.8602
2451	-2.1607	0.2669	-0.8600	1.1177	2.0291	1.8735
7475	-2.2814	0.3345	-0.9833	0.9948	2.1326	1.8875
22793	-2.3000	0.3500	-1.0146	0.9566	2.1500	1.8931

Table 1. Best individual of some generations.

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